Ma2a Practical – Recitation 1

Fall 2024

Exercise 1. Consider a first order linear differential equation $y' + p(t)y = q(t)$. Determine a general expression in terms of p and q for an integrating factor. Deduce the expression of a special solution to the equation.

Exercise 2. Solve the differential equation

$$t^2y' + y' + y = \frac{5t}{e^{\tan^{-1}(t)}}.$$

Exercise 3. Solve the differential equation $y' + p(t)y = 0$, where $p(t) = 2$ for $0 \leqslant t \leqslant 1$ and $p(t) = 1$ for $t > 1$.

Exercise 4. Draw the direction field of the equation

$$y' = y^2 + t^2.$$

Exercise 5. Consider the function $u(t)$ which satisfies the differential equation

$$\mu u' + \lambda u = e^t$$

subject to the initial condition $u(0) = u_0$. Show that $u(t) = v(t) + w(t)$ where $w(t)$ satisfies the same differential equation as $u(t)$ with initial value $w(0) = 0$ and $v(t)$ satisfies the corresponding homogenous equation $\mu v' + \lambda v = 0$ with initial value $v(0) = u_0$.